```
> restart; kernelopts(version); Digits:=18;
```
$$\text{Maple 12.02, IBM INTEL NT, Dec 10 2008 Build ID 377066}$$
$$Digits := 18$$

[+] nearest IEEE rational

```
> nearest:=proc(z)
  local x,y;
  Digits:=54;
  my_nearest_rational(Re(evalf(z))) + my_nearest_rational(Im(evalf(z)))*I;
  end proc;
```
$$nearest := \mathbf{proc}(z)\ \mathbf{local}\ x,\ y;\ Digits := 54;\ my\_nearest\_rational(\Re(evalf(z))) + my\_nearest\_rational(\Im(evalf(z)))*I\ \mathbf{end\ proc}$$

```
> 'nearest(-1 +1.0/2^50)': '%'=%;
  'nearest(-1 +1.0/2^52 + 1e-12*I)': '%'=%;
```
$$nearest(-0.999999999999999112) = -\frac{1125899906842623}{1125899906842624}\,pow(2,0)$$

$$nearest(-0.999999999999999778 + 0.1\ 10^{-11}\ I) = -\frac{4503599627370495}{4503599627370496}\,pow(2,0) + \frac{4951760157141521}{9007199254740992}\,I\,pow(2,-39)$$

[+] Calling the DLL from Maple

[−] Example discussed at Nuclear Phyance

```
> tstRoutine:=proc(Z)
    local z, gm,gd;
    forget(evalf);
    z:=nearest(Z);
    print(` z`[IEEE]= z);
    z:=eval(z); # now evaluate pow
    gd:=Gamma_DLL(z);
    evalf[200](GAMMA(z)): gm:=evalf(%);

    print(`DLL  `= gd);
    print(`exact`= gm);
    print(`error (absolute, relative)` = gm - gd, 1 - abs(gd/gm) );
  end proc:
>
> zTst:=-1.0+1/2^50;
  tstRoutine(zTst);
```
$$zTst := -0.999999999999999112$$
$$z_{IEEE} = -\frac{1125899906842623}{1125899906842624}\,pow(2,0)$$
$$DLL\ = -0.112589990684262450\ 10^{16}$$
$$exact = -0.112589990684262442\ 10^{16}$$
$$error\ (absolute,\ relative) = 0.08,\ -0.7\ 10^{-16}$$

```
> zTst:=-1.0+1/2^50 - I*1e-12;
  tstRoutine(zTst);
```
$$zTst := -0.999999999999999112 - 0.1\ 10^{-11}\ I$$
$$z_{IEEE} = -\frac{1125899906842623}{1125899906842624}\,pow(2,0) - \frac{4951760157141521}{9007199254740992}\,I\,pow(2,-39)$$
$$DLL\ = -0.888177719474230170\ 10^{9} - 0.999999211139717042\ 10^{12}\ I$$
$$exact = -0.888177719474230155\ 10^{9} - 0.999999211139717100\ 10^{12}\ I$$
$$error\ (absolute,\ relative) = 0.15\ 10^{-7} - 0.000058\ I,\ 0.58\ 10^{-16}$$

```
> zTst:=-4.0-1/2^50;
  tstRoutine(zTst);
```
$$zTst := -4.00000000000000089$$
$$z_{IEEE} = -\frac{4503599627370497}{9007199254740992}\,pow(2,3)$$
$$DLL\ = -0.469124961184426016\ 10^{14}$$
$$exact = -0.469124961184426039\ 10^{14}$$
$$error\ (absolute,\ relative) = -0.0023,\ 0.49\ 10^{-16}$$

```
> zTst:=-4.0-1/2^50 - I*1e-12;
  tstRoutine(zTst);
```
$$zTst := -4.00000000000000089 - 0.1\ 10^{-11}\ I$$
$$z_{IEEE} = -\frac{4503599627370497}{9007199254740992}\,pow(2,3) - \frac{4951760157141521}{9007199254740992}\,I\,pow(2,-39)$$

$$\text{DLL} = -0.370074048977220058 \cdot 10^8 + 0.416666337974882050 \cdot 10^{11} \, I$$

$$\text{exact} = -0.370074048977220063 \cdot 10^8 + 0.416666337974882125 \cdot 10^{11} \, I$$

$$\text{error (absolute, relative)} = -0.5 \cdot 10^{-9} + 0.75 \cdot 10^{-5} \, I, \ 0.180 \cdot 10^{-15}$$

```
> zTst:=(-18.0+1/2^52 + I* 1/2^52); #zTst:=evalf(%);
  tstRoutine(zTst);
```

$$zTst := -17.9999999999999998 + 0.222044604925031308 \cdot 10^{-15} \, I$$

$$z_{\text{IEEE}} = -\frac{9}{16} \, \text{pow}(2, 5) + \text{pow}(2, -52) \, I$$

$$\text{DLL} = 0.455751655153189450 \cdot 10^{-15} - 0.703426546835479560 \, I$$

$$\text{exact} = 0.455751655153187192 \cdot 10^{-15} - 0.703426546835475839 \, I$$

$$\text{error (absolute, relative)} = -0.2258 \cdot 10^{-29} + 0.3721 \cdot 10^{-14} \, I, \ -0.529 \cdot 10^{-14}$$

```
> 
```

More short extreme tests

```
> zTst:=evalhf(1.4-1/2^50);
  tstRoutine(zTst);
```

$$zTst := 1.39999999999999902$$

$$z_{\text{IEEE}} = \frac{3152519739159345}{4503599627370496} \, \text{pow}(2, 1)$$

$$\text{DLL} = 0.887263817503075592$$

$$\text{exact} = 0.887263817503075342$$

$$\text{error (absolute, relative)} = -0.250 \cdot 10^{-15}, \ -0.28 \cdot 10^{-15}$$

```
> zTst:=evalhf(1.4-1/2^50 - I* 1e-12);
  tstRoutine(zTst);
```

$$zTst := 1.39999999999999902 - 0.99999999999999998 \cdot 10^{-12} \, I$$

$$z_{\text{IEEE}} = \frac{3152519739159345}{4503599627370496} \, \text{pow}(2, 1) - \frac{4951760157141521}{9007199254740992} \, I \, \text{pow}(2, -39)$$

$$\text{DLL} = 0.887263817503075592 + 0.544642853642789336 \cdot 10^{-13} \, I$$

$$\text{exact} = 0.887263817503075342 + 0.544642853642787714 \cdot 10^{-13} \, I$$

$$\text{error (absolute, relative)} = -0.250 \cdot 10^{-15} - 0.1622 \cdot 10^{-27} \, I, \ -0.28 \cdot 10^{-15}$$

```
> zTst:=evalhf(1+1/2^50);
  tstRoutine(zTst);
```

$$zTst := 1.00000000000000088$$

$$z_{\text{IEEE}} = \frac{1125899906842625}{2251799813685248} \, \text{pow}(2, 1)$$

$$\text{DLL} = 0.999999999999999440$$

$$\text{exact} = 0.999999999999999487$$

$$\text{error (absolute, relative)} = 0.47 \cdot 10^{-16}, \ 0.47 \cdot 10^{-16}$$

```
> zTst:=evalhf(1-1/2^50 + I* 1e-12);
  tstRoutine(zTst);
```

$$zTst := 0.99999999999999912 + 0.99999999999999998 \cdot 10^{-12} \, I$$

$$z_{\text{IEEE}} = \frac{1125899906842623}{1125899906842624} \, \text{pow}(2, 0) + \frac{4951760157141521}{9007199254740992} \, I \, \text{pow}(2, -39)$$

$$\text{DLL} = 1. - 0.577215664901534186 \cdot 10^{-12} \, I$$

$$\text{exact} = 1.00000000000000051 - 0.577215664901534606 \cdot 10^{-12} \, I$$

$$\text{error (absolute, relative)} = 0.51 \cdot 10^{-15} - 0.420 \cdot 10^{-27} \, I, \ 0.510 \cdot 10^{-15}$$

```
> zTst:=evalhf(0-1/2^155); #zTst:=1.00000000000000088;
  tstRoutine(zTst);
```

$$zTst := -0.218952885050752668 \cdot 10^{-46}$$

$$z_{\text{IEEE}} = -\frac{1}{2} \, \text{pow}(2, -154)$$

$$\text{DLL} = -0.456719261665907162 \cdot 10^{47}$$

$$\text{exact} = -0.456719261665907162 \cdot 10^{47}$$

$$\text{error (absolute, relative)} = 0., \ 0.$$

```
> zTst:=evalhf(0+1/2^50 + I* 1e-12);
  tstRoutine(zTst);
```

$$zTst := 0.888178419700125232 \cdot 10^{-15} + 0.99999999999999998 \cdot 10^{-12} \, I$$

$$z_{IEEE} = pow(2, -50) + \frac{4951760157141521}{9007199254740992} I\ pow(2, -39)$$

$$DLL = 0.888177718474230170\ 10^9 - 0.999999211139716920\ 10^{12}\ I$$

$$exact = 0.888177718474230155\ 10^9 - 0.999999211139717100\ 10^{12}\ I$$

$$error\ (absolute,\ relative) = -0.15\ 10^{-7} - 0.000180\ I,\ 0.180\ 10^{-15}$$

```
> zTst:=(-1.0+1/2^50); #zTst:=1.00000000000000088;
  tstRoutine(zTst);
```

$$zTst := -0.999999999999999112$$

$$z_{IEEE} = -\frac{1125899906842623}{1125899906842624}\ pow(2, 0)$$

$$DLL = -0.112589990684262450\ 10^{16}$$

$$exact = -0.112589990684262442\ 10^{16}$$

$$error\ (absolute,\ relative) = 0.08,\ -0.7\ 10^{-16}$$

```
> zTst:=(-1.0-1/2^52); #zTst:=1.00000000000000088;
  tstRoutine(zTst);
```

$$zTst := -1.00000000000000022$$

$$z_{IEEE} = -\frac{4503599627370497}{9007199254740992}\ pow(2, 1)$$

$$DLL = 0.450359962737049550\ 10^{16}$$

$$exact = 0.450359962737049558\ 10^{16}$$

$$error\ (absolute,\ relative) = 0.08,\ 0.18\ 10^{-16}$$

```
> zTst:=(-1-1/2^50 + I* 1/2^40);
  tstRoutine(zTst);
```

$$zTst := \frac{-1125899906842625}{1125899906842624} + \frac{1}{1099511627776}\ I$$

$$z_{IEEE} = -\frac{1125899906842625}{2251799813685248}\ pow(2, 1) + \frac{1}{2}\ I\ pow(2, -39)$$

$$DLL = 0.107374079957819224\ 10^{10} + 0.1099510579201\ 10^{13}\ I$$

$$exact = 0.107374079957819223\ 10^{10} + 0.109951057920100000\ 10^{13}\ I$$

$$error\ (absolute,\ relative) = -0.1\ 10^{-7} + 0.\ I,\ 0.$$

```
> zTst:=(-1+1/2^51 - I* 3/2^40);
  tstRoutine(zTst);
```

$$zTst := \frac{-2251799813685247}{2251799813685248} - \frac{3}{1099511627776}\ I$$

$$z_{IEEE} = -\frac{2251799813685247}{2251799813685248}\ pow(2, 0) - \frac{3}{4}\ I\ pow(2, -38)$$

$$DLL = -0.596523223980930150\ 10^8 - 0.366503866216296510\ 10^{12}\ I$$

$$exact = -0.596523223980930189\ 10^8 - 0.366503866216296553\ 10^{12}\ I$$

$$error\ (absolute,\ relative) = -0.39\ 10^{-8} - 0.000043\ I,\ 0.117\ 10^{-15}$$

```
> zTst:=evalhf(-1.4+1/2^50); #zTst:=1.00000000000000088;
  tstRoutine(zTst);
```

$$zTst := -1.39999999999999902$$

$$z_{IEEE} = -\frac{3152519739159345}{4503599627370496}\ pow(2, 1)$$

$$DLL = 2.65927187288003442$$

$$exact = 2.65927187288003489$$

$$error\ (absolute,\ relative) = 0.47\ 10^{-15},\ 0.177\ 10^{-15}$$

```
> zTst:=evalhf(-1.4+1/2^50 + I* 1e-12);
  tstRoutine(zTst);
```

$$zTst := -1.39999999999999902 + 0.99999999999999998\ 10^{-12}\ I$$

$$z_{IEEE} = -\frac{3152519739159345}{4503599627370496}\ pow(2, 1) + \frac{4951760157141521}{9007199254740992}\ I\ pow(2, -39)$$

$$DLL = 2.65927187288003442 + 0.445073424907542654\ 10^{-11}\ I$$

$$exact = 2.65927187288003489 + 0.445073424907542752\ 10^{-11}\ I$$

$$error\ (absolute,\ relative) = 0.47\ 10^{-15} + 0.98\ 10^{-27}\ I,\ 0.177\ 10^{-15}$$

```
> zTst:=(-8+1/2^50);
  tstRoutine(zTst);
```

$$zTst := \frac{-9007199254740991}{1125899906842624}$$

$$z_{IEEE} = -\frac{9007199254740991}{9007199254740992}\,pow(2, 3)$$

$$DLL = 0.279241048324064026\ 10^{11}$$

$$exact = 0.279241048324064023\ 10^{11}$$

$$error\ (absolute,\ relative) = -0.3\ 10^{-6},\ -0.1\ 10^{-16}$$

```
> zTst:=(-2+1/2^52 + I* 1/2^52); #zTst:=evalhf(zTst);
  tstRoutine(zTst);
```

$$zTst := \frac{-9007199254740991}{4503599627370496} + \frac{1}{4503599627370496}\,I$$

$$z_{IEEE} = -\frac{9007199254740991}{9007199254740992}\,pow(2, 1) + \frac{1}{2}\,I\,pow(2, -51)$$

$$DLL = 0.1125899906842624750\ 10^{16} - 0.1125899906842624\ 10^{16}\,I$$

$$exact = 0.1125899906842624462\ 10^{16} - 0.1125899906842624000\ 10^{16}\,I$$

$$error\ (absolute,\ relative) = -0.29 + 0.\ I,\ -0.13\ 10^{-15}$$

```
> zTst:=(-3+1/2^52 + I* 1/2^52); #zTst:=evalhf(zTst); zTst:=fnormal(zTst,16); zTst:=convert(zTst,
  rational);
  tstRoutine(zTst);
```

$$zTst := \frac{-13510798882111487}{4503599627370496} + \frac{1}{4503599627370496}\,I$$

$$z_{IEEE} = -\frac{3}{4}\,pow(2, 2) + \frac{1}{2}\,I\,pow(2, -51)$$

$$DLL = -0.209352944738633416 + 0.750599937895082625\ 10^{15}\,I$$

$$exact = -0.209352944738633412 + 0.750599937895082667\ 10^{15}\,I$$

$$error\ (absolute,\ relative) = 0.4\ 10^{-17} + 0.042\ I,\ 0.56\ 10^{-16}$$

```
> zTst:=(-18+1/2^52 + I* 1/2^52); #zTst:=evalhf(zTst); zTst:=fnormal(zTst,16); zTst:=convert(zTst,
  rational);
  tstRoutine(zTst);
```

$$zTst := \frac{-81064793292668927}{4503599627370496} + \frac{1}{4503599627370496}\,I$$

$$z_{IEEE} = -\frac{9}{16}\,pow(2, 5) + \frac{1}{2}\,I\,pow(2, -51)$$

$$DLL = 0.455751655153189450\ 10^{-15} - 0.703426546835479560\,I$$

$$exact = 0.455751655153187192\ 10^{-15} - 0.703426546835475839\,I$$

$$error\ (absolute,\ relative) = -0.2258\ 10^{-29} + 0.3721\ 10^{-14}\,I,\ -0.529\ 10^{-14}$$

```
> zTst:=(-118+1/2^12 + I* 1/2^52); #zTst:=evalhf(zTst); zTst:=fnormal(zTst,16); zTst:=convert(zTst,
  rational);
  tstRoutine(zTst);
```

$$zTst := \frac{-483327}{4096} + \frac{1}{4503599627370496}\,I$$

$$z_{IEEE} = -\frac{483327}{524288}\,pow(2, 7) + \frac{1}{2}\,I\,pow(2, -51)$$

$$DLL = 0.875388082539475864\ 10^{-191} - 0.795325421020013932\ 10^{-203}\,I$$

$$exact = 0.875388082539550118\ 10^{-191} - 0.795325421020081427\ 10^{-203}\,I$$

$$error\ (absolute,\ relative) = 0.74254\ 10^{-204} - 0.67495\ 10^{-216}\,I,\ 0.84824\ 10^{-13}$$

```
> zTst:=(-118+1/2^12 - I* 95); #zTst:=evalhf(zTst); zTst:=fnormal(zTst,16); zTst:=convert(zTst,
  rational);
  tstRoutine(zTst);
```

$$zTst := \frac{-483327}{4096} - 95\,I$$

$$z_{IEEE} = -\frac{483327}{524288}\,pow(2, 7) - \frac{95}{128}\,I\,pow(2, 7)$$

$$DLL = 0.$$

$$exact = -0.184969398699680654\ 10^{-308} - 0.374899984400821005\ 10^{-308}\,I$$

$$error\ (absolute,\ relative) = -0.184969398699680654\ 10^{-308} - 0.374899984400821005\ 10^{-308}\,I,\ 1.$$